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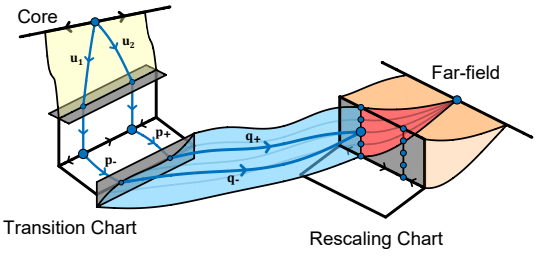
MATHEMATICAL INTERESTS

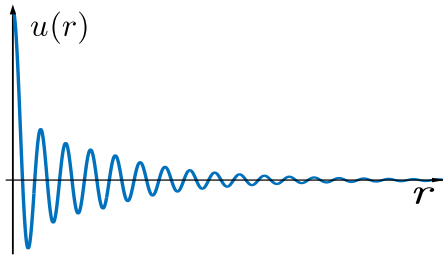
Dynamical systems, pattern formation, bifurcation theory, localised structures, spatial dynamics, radial problems.

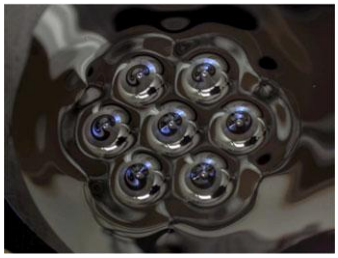
EDUCATION

- **PhD Applied Mathematics.** University of Surrey
  Title: Existence of Small Amplitude Localised Patterns on the Surface of a Ferrofluid *October 2017 - Present*
  - **Supervisors**: David Lloyd and Matthew Turner.
  - **Submitted**: 31st March 2021.
  - **Research**: Within the mathematics department, I am a member of the 'Nonlinear waves and geophysical fluid dynamics' and 'Dynamical systems and partial differential equations' research groups. My current research focuses around the existence of localised patterns; in particular, I investigated localised radial patterns on the surface of a ferrofluid, a magnetic fluid consisting of iron nanoparticles. In 2005, for a vertically applied magnetic field, localised radial spikes were experimentally observed emerging from the flat state.

    The ferrofluid experiment can be formulated as a free-surface problem, which we expressed as a quasilinear PDE system that is non-autonomous in the radial coordinate r. We constructed an r-independent basis formed from eigenmodes of the linear operator in the far-field; projecting onto each respective mode reduces the PDE system to an infinite set of nonlinear radial ODEs. Using the theory of stable foliations and local invariant manifolds, we constructed local manifolds in the core and far-field regions, containing all solutions that remain bounded as $r \to 0$ and decay exponentially as $r \to \infty$, respectively. Using geometric blow-up coordinates seen for the Swift-Hohenberg equation (SHE) and looking for intersections of the core and far-field manifolds (Figure, left), we identified three classes of localised radial patterns and determined their radial profiles. Notably, these solutions correspond exactly to the localised radial patterns found in the SHE and one of these solutions, known as spot A (Figure, centre), is believed to be the spot observed in the ferrofluid experiment.







    Another interesting phenomena is the emergence of localised cellular patterns; such as rhomboids, square, and hexagons, for example. Localised hexagons have been observed bifurcating from the radial spot in the ferrofluid experiment (Figure, right), and a large variety of localised cellular patterns have been found numerically and experimentally in other areas of research; for example, in nonlinear optics.

    We have recently been investigating the existence of localised cellular patterns in the SHE, since it represents one of the simplest pattern-forming systems. We approximated the full planar SHE by a truncated Fourier series in polar coordinates; by projecting onto each Fourier mode, we obtain a finite-dimension coupled system of nonlinear radial ODEs. Then, extending the techniques seen for radial solutions, we again construct core and far-field manifolds and look for intersections. In this case, localised solutions are subject to an algebraic matching condition that depends on the truncation order N and lattice m imposed by our Fourier decomposition. By solving the small truncation matching problems ($N = 1, 2, 3$), we are able to use numerical continuation codes in MATLAB to find larger localised cellular patterns in parameter space.
  - **Training**: I attend the mathematics department colloquia as well as seminars within my research groups. Throughout my PhD, I have undertaken several taught courses including Dynamical Systems, PDEs, Mathematical Methods, and Functional Analysis. I have also attended several workshops on teaching and assessments.

- **MMath Mathematics** University of Surrey
  Degree class: First - 86% *September 2013 - June 2017*
  - **Project**: I completed my MMath project under the supervision of Dr Bin Cheng (University of Surrey), rigorously investigating solutions to the Navier-Stokes equations in a thin spherical shell. This involved a careful application of functional analysis in a small, non-convex domain, in order to identify analytical problems to overcome in future work.

Awards & Grants

2021 Finalist for the IMA Lighthill Thwaites Prize
2020 LMS Early Research travel grant, value £500, London Mathematical Society*
— FEPS travel award, value £850, Faculty of Engineering and Physical Sciences, University of Surrey*
2019 FEPS travel award, value £770, Faculty of Engineering and Physical Sciences, University of Surrey
**Unclaimed due to the COVID-19 pandemic*

Publications

Dan J. Hill, David J.B. Lloyd & Matthew R. Turner, *Localised Radial Patterns on the Free Surface of a Ferrofluid*, accepted J. Nonlinear Sci. (2021), arXiv Link*

**Shortlisted for the IMA Lighthill Thwaites Prize 2021*

Publications in preparation

Dan J. Hill & David J.B. Lloyd, *Existence of Approximate Localised Cellular Patterns*
Dan J. Hill, *Localised Radial Patterns in a Model for Dryland Vegetation*

Research Presentations

Seminars:	Invited	Leeds Applied Nonlinear Dynamics (LAND) Seminars, **University of Leeds**
	2020	Waves in One World, **One World Seminars**
	—	Chair for Analysis and Modelling Seminar, **University of Stuttgart**
	—	Applied Analysis Seminar, **University of Bremen**
	—	DSPDEs & NWGFD Research Seminar, **University of Surrey**
	2019	Applied Analysis Seminar, **Saarland University**
Conferences:	Invited	Contributed Talk, **SIAM DS21**; Oral Presentation, **ICTAM 2020+1**
	2021	IMA Lighthill Thwaites Prize Minisymposium, **BAMC 2021**
	2020	'Wave and front dynamics' Minisymposium, **SIAM NWCS20***
	2019	Poster Presentation, **Equadiff 2019**
	—	Contributed Talk, **BAMC 2019**
		**Conference cancelled due to the COVID-19 pandemic*

Roles & Responsibilities

- **Teaching experience**: I have consistently provided **marking support** across my PhD, assessing a variety of subjects that range from calculus and differential equations, to mathematical modelling and Hamiltonian dynamics. In more recent years, I have also gained experience in a **teaching assistant**-role; giving one-to-one support in tutorials for ordinary differential equations, as well as leading large-class tutorials for linear partial differential equations.
- **Outreach**: Increasing representation and battling discrimination in mathematics are both important causes that I have tried to support throughout my career. To this end, I have served as a member of the departmental **Athena Swan committee**, whose mission is to tackle gender inequality in STEM subjects, as well as being involved in a number of outreach activities aimed at underrepresented groups. As part of these events, I: prepared and presented talks, ran workshops and visited local schools. In particular, I helped lead a collection of group projects as part of the **Widening Participation Summer School** (2018 & 2019) for A-Level students. As part of these events I prepared various research projects, led an introductory workshop, and provided one-to-one support to the students throughout the week.
- **Departmental Roles**: I have undertaken a variety of roles during my PhD; I developed leadership and support skills throughout my tenure as **Postgraduate Student Rep** and as a **student mentor**, as well as filling a number of administrative positions among the PhD students. These range from social events– such as hosting a football sweepstake and a departmental quiz– to also founding two series of postgraduate-run seminars: the **Postgraduate Research Seminars** and the **'Taste of Research' Undergraduate Seminars (TORUS)**. I organised and chaired these seminars for two years, and they continue to be run by other PhD students in the department. I also co-organised a **conference minisymposium**, titled *'Localisation in Physical and Biological Applications'*, for SIAM NWCS20* in partnership with Mark Groves (Saarland).
  **Conference cancelled due to the COVID-19 pandemic*